Simultaneous detection and recognition of chromatic flashes*

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Studies of simultaneous detection and recognition were performed to test alternative models of the detection process, signal detection theory and low-threshold theory. Sensitivity in a detection experiment was independent of whether the type of signal (red or green light flash) was known in advance, because only one type of s trial was possible, or was unknown, because either stimulus could occur. When a recognition judgment was added to either a binary or rating-scale detection response, Ss were able to report the nature of the stimulus at better than chance levels even when they indicated that the stimulus was not detected. Since such performance occurred when Ss used detection responses likely to have been given only in the nondetect state, the data lead to the rejection of low-threshold theory.

In recent years, several models have been developed which embody different conceptions of the nature of the detection process (Green & Swets, 1966), and a considerable amount of theoretical and empirical work has been expended in efforts to decide between them. The classical high-threshold theory has most often been contrasted with two contemporary theories, the theory of signal detectability of Swets, Tanner, Green, & others (Swets, 1964) and the low-threshold theory of Luce (1963a, b). High-threshold theory assumes that there exists a fixed threshold which is exceeded on some proportion of trials during which a stimulus occurs (s trials), but is never passed when no stimulus is presented (n trials). Considerable evidence has been marshalled against this conception of the threshold (Swets, 1964).

A comparison of the two contemporary theories is not as easy. They share certain characteristics, namely that internal or external "noise" can sometimes cause neural effects which lead the O to believe a stimulus had been presented on trials when in fact there was none, and that suitable motivational variables can cause an O to change the proportion of s and n trials on which he will report the presence of a stimulus.

The two-state theory of Luce retains the concept of a threshold, but one which is assumed to be passed by noise on some percentage of n trials; hence, the designation "low-threshold theory." Luce's theory conceives that following each trial, the O is in one of two states, detect or nondetect, depending on whether his sensory threshold was exceeded or not. Furthermore, it assumes that motivational variables, such as probability of signal presentation or payoffs for correct decisions, determine the probability, conditioned on his state, that the O will report the presence or absence of the stimulus.

According to signal detection theory, no fixed threshold exists. Rather, there is a continuum of sensory activation along which an O can establish a criterion. This criterion is movable, and its location is determined by the presentation probability of the signal and the O's instructions. The theory conceives of two overlapping normal distributions, one representing the probability density of sensory magnitude given the presentation of noise alone and the other representing the probability density given signal plus noise. The criterion will be exceeded on some proportion of n trials (thus giving rise to the false-alarm rate), but on a larger proportion of s trials.

The experimental paradigms of these theories generally involve presenting large numbers of s and n trials and noting the proportion of each type of trial on which the O reports the presence of the stimulus, $P(Y \mid s)$ and $P(Y \mid n)$, otherwise known as hit and false-alarm rates, respectively. Changes in payoffs or presentation probabilities make it possible to obtain several different pairs of values of $P(Y \mid s)$ and $P(Y \mid n)$. Receiver-operating-characteristic (ROC) curves can be plotted from these data, with $P(Y \mid n)$ as the abscissa and $P(Y \mid s)$ as the ordinate.

The two theories lead to different predictions regarding the form of the ROC curve. According to signal detection theory, it is a continuous function. According to low-threshold theory, the ROC curve consists of two straight lines, provided the O uses both of the responses ("no" and "yes") when he is in one of the two states (nondetect or detect). The two lines will intersect at a point whose coordinates correspond to the proportion of s and n trials on which the threshold was passed. Unfortunately, the theoretical functions predicted by the two theories are sufficiently similar so that the theories, even though their underlying assumptions are very different, cannot be tested readily by their ability to fit experimentally obtained sets of hit and false-alarm rates.

Signal detection theory does not limit the number of criteria that an O can hold to only one at a time. An O is assumed capable of having a number of criteria simultaneously, so that he can use more than two response alternatives following each trial. These response alternatives can be treated as ratings of the likelihood that a stimulus was presented, or of the magnitude of the sensory activity occurring on each trial. A two-state theory like Luce's assumes that in each state, the O cannot make any further relevant discriminations. On the first theory, it should be possible to discern a continuous ordering of the a posteriori probabilities of a signal given the different response categories. On the second theory, no continuous ordering should exist, provided that the O uses more than one response category only when he is in one of the two states. Nachmias and Steinman (1963) were the first to examine these predictions with visual data and found support for signal detection theory. Considerable theoretical discussion of this matter ensued (Larkin, 1965; Watson & Bourbon, 1965; Broadbent, 1966; Wickelgren, 1968), and the current situation is well reviewed by Krantz (1969).

However, an alternative basis for testing low-threshold theory and signal detection theory is available. Suppose there are two possible stimuli, A and B, which can be presented on an s trial. An O can be required to make two responses—a detection response (the stimulus was or was not present) and a recognition response (the stimulus was A or B). Even on trials for which he believes no stimulus was presented, the O can be asked to make a judgment as to whether he believes A or B was more likely to have occurred if there actually had been a stimulus.

Signal detection theory indicates that an O says "no" whenever the observation on that trial falls below

*This research was supported by United States Public Health Service Grant NB-03682 to Jacob Nachmias and by Fight for Sight Student Fellowship SF-262 from Fight for Sight, Inc., to Gary B. Rollman. Preparation of this paper was aided by Grant APA-392 to Gary B. Rollman from the National Research Council of Canada.

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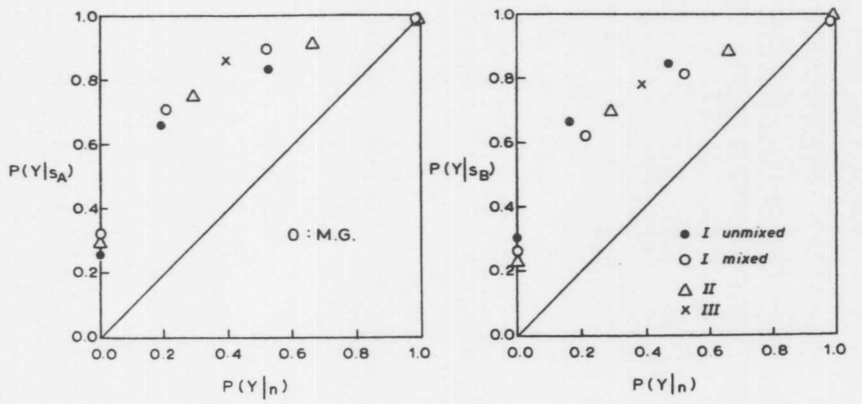

Fig. 1. ROC plots describing the detection of red (s_A) and green (s_B) light flashes in the unmixed and mixed sessions of Experiment I and the mixed sessions of Experiments II and III. Os employed a 5-point rating scale in Experiments I and II and a yes-no response in Experiment III. In the latter two experiments, recognition judgments were made in addition to the detection responses. The 5-point scale yielded 4 points for each of the conditions in Experiments I and II; one of these, in each condition, was so close to the upper right-hand corner of the figure that the point may be obscured.

the criterion he has established. However, even such observations may contain enough information to enable the O to discriminate between Stimuli A and B at a level better than chance. Therefore, even when the O says "no," his recognition responses may be distributed nonrandomly among the different types of trials.

On the other hand, two-state threshold theories lead us to expect that all trials on which insufficient neural activity was generated to exceed threshold, should be indistinguishable. Specifically, on trials where the O emits a detection response which he employs only in the nondetect state, his recognition responses should not depend upon which stimulus, if any, was presented.

The first experiment of this type was done by Shipley (1965), who used auditory tone bursts of two frequencies. Her results conformed with threshold-theory expectations. However, in a more recent study by Lindner (1968), who also used auditory sinusoids, results were consistent with signal detection theory. Lindner ascribed Shipley's results to possible biasing effects caused by assumptions made by her Os when faced with the detection and recognition task. In the only relevant visual experiment reported to date, Munsinger and Gummerman (1968) presented Os with vertical and horizontal bars flashed by a tachistoscope. They varied payoffs to obtain different false-alarm rates and observed that the probability of correct recognition, given a hit, decreased as the false-alarm rate increased. However, they failed to examine closely the more important question: What is the recognition performance, given a miss, that is obtained with different false-alarm rates?

The present study used the simultaneous detection and recognition paradigms to explore that problem. Dim red and green flashes were presented to both dark- and light-adapted Os and either yes-no or rating-scale detection responses were employed. Data from such experiments should help determine whether a sensory threshold exists for the detection of flashes of light.

METHOD

Stimulus Conditions

The O looked with his right eye through a 1.0-mm artificial pupil into a conventional multichannel Maxwellian-view system which has been described previously (Nachmias & Steinman, 1963). He saw a red fixation dot, made with a Wratten 25A filter, in a field which was totally dark. The stimulus to be detected was a disk 10 min in diam, centered 26.2 min above the fixation point and flashed for 43 msec. The color of the disk was either red (dominant wavelength 633 nm, purity 100%) or green (dominant wavelength 501 nm, purity 75%), depending upon whether Wratten Color Filter 29 or 65 was interposed in the light path. The color filters were combined with neutral density filters in an effort to render the green and red flashes equally detectable. The necessary neutral filter densities were determined from the results of some preliminary experiments in which psychometric functions were measured by the method of constant stimuli.

One O (R.A.) was run under slightly different stimulus conditions, since he saw the light flashes superimposed upon a white background. The 10-min disk was centered on a 1-deg circular field whose luminance was 1.106 log mL. A reticle with fine cross hairs intersecting at the center of the background field served in place of the red dot as a fixation reference.

Observers

Six men served as Os in some or all of these experiments: R.S. was a research associate with considerable experience in psychophysical experiments; R.A., J.E., J.G., M.G., and V.V. were undergraduates with no previous experience, who were paid an hourly wage for their participation.

All Os were tested on a Nagel anomaloscope and were found to have normal color vision. None knew the purpose of these experiments. All were given considerable practice on relevant psychophysical tasks.

Procedure

Common to all experiments. Each experimental session began with 15 min of dark adaptation in total darkness. When the background field was to be used, the O was light adapted to its luminance for a minute or so before any observations were made.

Every session consisted of 420 trials, divided by rest periods into 12 blocks of 35 trials each. Trials occurred at 10-sec intervals. Each trial began with a warning buzzer, followed after a little over a second by the click of a solenoid shutter. During the click, either the red flash (s_A trial) or the green flash (s_B trial) or nothing (n trial) was presented. The O was required to make a detection response (yes-no, or rating scale, as discussed later) after every trial. In Experiments II and III, he was also required to make a recognition response (A or B) on every trial. At the end of each trial, O was told whether or not a signal had been presented, and in addition, in Experiments II and III, he was told whether s_A or s_B had been presented.

At the start of every experimental session, the O was shown examples of the types of trials he would encounter. When the two test flashes were shown, they were referred to only as A and B and no color names were used, so as to avoid giving the Os the impression that the flashes would necessarily look red and green.

Specific Procedure

Experiment I: Mixed and unmixed detection. Mixed and unmixed detection were investigated on alternate experimental sessions. In mixed detection sessions, the three

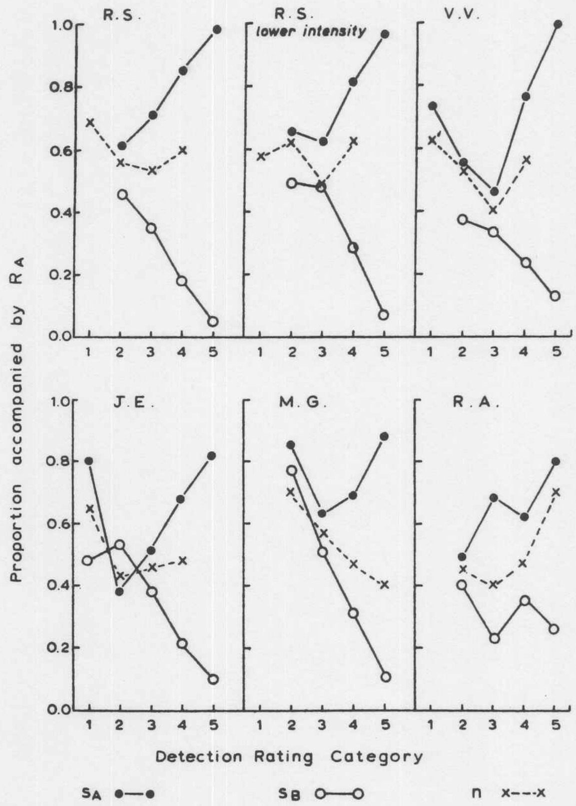

Fig. 3. Proportion of detection responses of each rating category in Experiment II that were recognized as s_A, with type of trial, s_A, s_B, or n, as a parameter.

types of trials (s_A, s_B, n) appeared approximately equally often and approximately at random in each of the 12 blocks of the session. In unmixed detection sessions, only one of the flashes (s_A or s_B) was presented on a random 2/3 of the trials during the first six blocks, while only the other flash was presented during the second 6 blocks of trials. In neither type of session were Os asked to make a recognition response, nor did they receive trial-by-trial feedback regarding the identity of the flashes. The Os were asked to use a 5-point rating scale for their detection responses, with R_5 signifying the highest certainty that a flash had been presented. They were told to use the five response categories in such a way that R_5 would be very rarely, if ever, given on catch trials and that R_1 would be given less often than R_2 on signal trials. An effort was thus made to give instructions about the use of the rating scale that would not be prejudicial to either detection theory under test.

Experiment II: Simultaneous rating-scale detection and recognition. All the sessions of this experiment were like the mixed detection sessions of the previous experiment. However, the Os were required to make a recognition response (R_A or R_B) on each trial, regardless of which detection response they used. They were instructed to make recognition responses with care, even when they believed no flash had been presented, since their belief might be wrong. Furthermore, they received trial-to-trial feedback about the identity, as well as the presence, of the flash.

Experiment III: Simultaneous yes-no detection and recognition. The procedure was the same as in the previous experiment, except that a simple yes-no detection task was substituted for the rating scale. The Os were asked to say "no" only when they were quite certain no flash had been presented, even if they had to make a great many false-alarms in the process. To bolster these instructions, they were given the task of winning as many points as possible on the basis of the following payoff matrix:

	R_yR_A	R_yR_B	R_nR_A	R_nR_B
s_A (red)	+2	0	0	−2
s_B (green)	0	+2	−2	0
n (blank)	−1	−1	+1	+1

At the end of each block of trials, the Os were told how many points they had won or lost. The points were not converted to any tangible goods or tokens.

RESULTS

Experiments designed to find out whether Os are able to recognize stimuli they claim not to detect necessarily involve more stimulus and response alternatives than do simple detection or recognition experiments. We shall therefore first consider whether overall detection and recognition performance is affected by the complexity of the stimulus-response structure of our experiments.

Detection Performance

The overall detection performances of two of our Os are summarized as standard ROC plots in Figs. 1 and 2. Responses in the different rating categories were cumulated in the manner prescribed by signal detection theory. For these and subsequent analyses, responses were pooled across several experimental sessions (three or four per condition in Experiment I and six in Experiments II and III).

The form of the ROC curves generated by Os M.G. and R.A. in Experiments I and II are quite typical

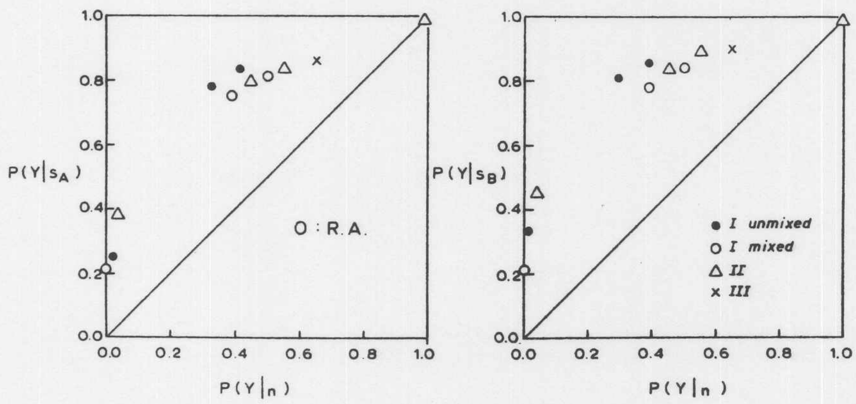

Fig. 2. ROC plots describing the detection of red (s_A) and green (s_B) light flashes in Experiments I, II, and III.

Table 1
Recognition Responses Accompanying Different Detection Responses: Experiment II

O	Type of Trial	"3"-"5" Responses Total	"3"-"5" Responses Proportion R_A	"1"-"2" Responses Total	"1"-"2" Responses Proportion R_A	χ^2***
RS	s_A (Red)	726	.86	111	.59	4.6*
	s_B (Green)	722	.17	121	.45	
	n	321	.56	519	.57	6.3*
RS+	s_A (Red)	481	.78	106	.65	6.2*
	s_B (Green)	432	.32	125	.49	
	n	220	.54	316	.62	8.1*
VV	s_A (Red)	710	.75	132	.55	6.7**
	s_B (Green)	762	.23	107	.37	
	n	382	.42	427	.53	8.5*
JE	s_A (Red)	730	.63	114	.42	2.5
	s_B (Green)	717	.26	128	.52	
	n	567	.47	264	.47	2.5
MG	s_A (Red)	812	.75	74	.86	2.7
	s_B (Green)	755	.35	103	.77	
	n	516	.58	260	.70	8.8*
RA	s_A (Red)	735	.70	136	.49	1.5
	s_B (Green)	730	.30	102	.41	
	n	447	.48	370	.45	1.5

*$.01 \leq p(\chi^2) \leq .05$ **$p(\chi^2) \leq .01$ +lower intensity flashes

***For each O, the first χ^2 value refers to a test of independence of R_A proportions between s_A and s_B trials; the second value refers to a test of these proportions obtained on s_A, s_B, and n trials.

of those from other Os in this and previous visual detection experiments. When plotted on double Gaussian coordinates, the points show no systematic deviation from a straight line with slope less than unity. On the usual SDT interpretations, both noise and signal-plus-noise distributions are Gaussian, with the variance of the latter being the greater.

The possible effect of signal uncertainty can be seen by comparing results from mixed sessions of Experiment I (where both s_A and s_B were presented in each block of trials) and from unmixed sessions (where either s_A or s_B, but not both, were presented for six blocks). O R.A. (Fig. 2) showed the largest and most consistent effect of stimulus uncertainty, in the expected direction. Both the red and the green flashes were better detected in the unmixed sessions. For O M.G. (Fig. 1), the effects are smaller and less consistent. Altogether, five Os were run in Experiment I. The results from each O were plotted on double Gaussian coordinates, and these plots were used to estimate d' [$d' = (\mu_s - \mu_n)/\sigma_n$] and σ_s/σ_n. The average difference between these estimates of d' from mixed and unmixed sessions was less than 0.01, which is less than our error of estimate of d'. Estimates of σ_s/σ_n differed in a more systematic manner. For four out of the five Os, the σ_s/σ_n ratio was greater in the mixed sessions (mean values of 1.31 and 1.53 in unmixed and mixed sessions, respectively).

In Experiment I, Os only had to report, by means of a 5-point rating scale, whether or not a flash had been presented; in Experiment II, they had to report, in addition, which of the two flashes had been presented. Inspection of Figs. 1 and 2 shows that the introduction of the recognition task led to no obvious deterioration in detection performance. If anything, for most Os, detection performance in Experiment II was somewhat better than in Experiment I, possibly because the latter was always run first. For one O (R.S.), the improvement was so marked that he had to be rerun in Experiment II with both s_A and s_B attenuated by an additional 20.6% (0.1 log unit).

The number of response alternatives was reduced in Experiment III by replacing the rating scale with a yes-no detection task. For O R.A., the point representing the hit and false-alarm rates generated in Experiment III lies along the ROC curve that could be drawn through the data of Experiment II. For O M.G., and for two of the others, the point from Experiment III lies slightly above the ROC curves from Experiment II. However, the improvement in detection performance was only slight, and may well have been due to the extra practice afforded by the second experiment.

Recognition Performance

In Experiment II, the five Os yielded overall correct recognition scores of 70%, 73%, 66%, 70%, and 68%. In Experiment III, their scores were 69%, 67%, 69%, 74%, and 65%—an average *deterioration* of a trivial 0.6%.

Relation Between Detection and Recognition

The primary concern in these experiments was the relationship between the recognition and detection responses from the same trials, and in particular, the recognition responses given on those trials when the Os asserted that no stimulus had been presented. Provided such detection responses are emitted only when the O is in the nondetect state, Luce's low-threshold theory predicts that the accompanying recognition responses will be distributed randomly among the three types of trial (s_A, s_B, n). From signal detection theory, on the other hand, we expect that these recognition responses may well be distributed nonrandomly.

Figure 3 shows the proportions of detection responses of each rating category in Experiment II that were accompanied by an R_A recognition response, with type of trial as parameter. Only proportions based on a total of more than 10 detection responses are plotted. Clearly, when O is quite certain a flash was presented and consequently uses the high end of the detection rating scale, his recognition response is usually correct (though even his "4s" are accompanied by recognition errors 15% to 30% of the time). Recognition accuracy is markedly inferior at the lower end of the detection rating category, though, on the whole, even "2s" are accompanied by a slight majority of correct recognition responses.

Low-threshold theory predicts that all trials on which the sensory threshold was not exceeded will be otherwise indistinguishable to the O. To test this prediction, it is necessary to examine a set of trials on which the detect state never occurred. On the basis of the instructions given to the Os and of the appearance of ROC plots such as Figs. 1 and 2, it is plausible to suppose that, if the theory were correct, R_1 and R_2 detection responses would only have been given in the nondetect state; all other response categories were used to report the detect state at least some of the time. Table 1 shows the results of pooling all R_1 and R_2 trials together and all R_3, R_4, and R_5 trials together. Note that all Os except J.E. use Recognition Response A more often when s_A (red flash) had been presented than when s_B (green flash) had been presented—even on trials eliciting detection responses "1" or "2." Chi-square tests of independence were applied separately to each O's recognition responses on R_1, R_2 trials. The hypothesis that R_A and R_B were given equally often on s_A, s_B, and n trials could be rejected beyond the 5% level of confidence in 4 out of 6 cases.

Excluding catch trials from the analyses reduced the number of significant chi-square values by one.

The results of Experiment III, summarized in Table 2, are quite similar. In that experiment, Os made only a binary detection decision. Instructions and a payoff matrix were intended to ensure that if low-threshold theory were true, "no" responses would be given only in the nondetect state. In consequence, "no's" were given rarely; even on catch trials, the maximum percentage of "no" responses was 63% while on s trials it was 26%. ROC plots of the resulting hit and false-alarm rates show them to be distinctly "upper limb" points. Yet in every case, even "no" responses are more often accompanied by Recognition Response "A," R_A, on s_A trials than on s_B trials. However, the differences in individual O's R_A rates are generally somewhat smaller here than in Experiment II, and less often statistically significant.

Taken together, however, the results of the two experiments indicate quite clearly that for our Os, discrimination performance was nonrandom, even when they claimed that no stimulus had been presented.

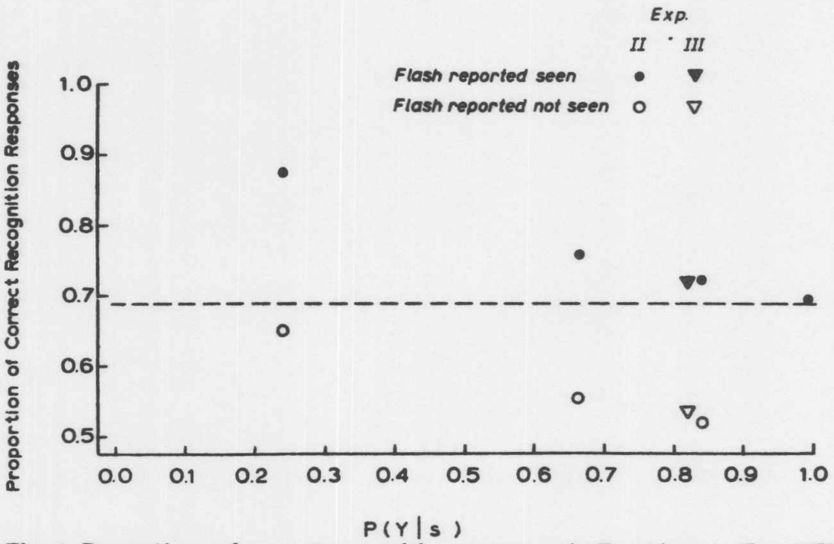

Fig. 4. Proportions of correct recognition responses in Experiments II and III as a function of detection criterion. The horizontal line represents the overall proportion of correct recognition responses, independent of which detection response was made.

DISCUSSION

Simultaneous detection and recognition experiments confront Os with a rather complex set of tasks, especially when a rating scale is employed. This complexity might in itself engender undesirable side effects such as inattention and guessing, which would preclude straightforward interpretation of the data obtained. We were, therefore, gratified to discover that, for our Os, detection performance did not appear to be materially affected by (1) whether one or two different flashes were presented within a block of trials, (2) the introduction of the recognition task, or (3) the substitution of a binary detection task for the rating scale. Recognition performance was also unaffected by the substitution of the binary detection task for the rating scale. In contrast, very different results have been obtained in auditory studies: the detection performance of two of Lindner's (1968) four listeners was drastically reduced when they also had to make recognition judgments, and in psychoacoustics, signal uncertainty consistently impairs detection performance (i.e., Creelman, 1960; Shipley, 1965).

Table 2
Recognition Responses Accompanying Different Detection Responses: Experiment III

O	Type of Trial	"Yes" Responses: Total	"Yes" Responses: Proportion R_A	"No" Responses: Total	"No" Responses: Proportion R_A	χ^2**
RS	s_A (Red)	450	.80	125	.58	1.1
	s_B (Green)	425	.32	150	.51	
	n	202	.62	328	.66	10.1*
VV	s_A (Red)	686	.58	159	.61	1.5
	s_B (Green)	725	.19	132	.54	
	n	305	.56	513	.59	1.7
JE	s_A (Red)	716	.67	159	.50	7.8*
	s_B (Green)	683	.24	142	.34	
	n	440	.48	380	.38	9.4*
MG	s_A (Red)	778	.72	128	.64	1.5
	s_B (Green)	642	.14	180	.57	
	n	312	.37	480	.58	1.8
RA	s_A (Red)	723	.66	119	.48	0.9
	s_B (Green)	770	.32	87	.41	
	n	533	.51	288	.48	1.3

*$p(\chi^2) \leq .01$
**For each O, the first χ^2 value refers to a test of independence of R_A proportions between s_A and s_B trials; the second value refers to a test of these proportions obtained on s_A, s_B, and n trials.

Our major finding is that recognition performance remains above chance level, even when Os are quite confident no signal has been presented. In this regard, our results substantiate those of Lindner (1968), even though his pure tones were more discriminable than were our chromatic flashes. Compare Fig. 2 in his paper with our Fig. 4; we have plotted proportion of recognition responses correct when Os reported the presence, and the absence, of the signal, as a function of detection criterion. The points and triangles represent means of Experiments II and III, respectively.

Lindner's findings and our own on the relation between recognition and detection are compatible in a general way with signal detection theory: neural activity that is insufficient to exceed a detection response criterion may well contain enough information for better than chance recognition. On the other hand, these findings clearly disagree with the most straightforward interpretation of low-threshold, two-state theory: recognition performance can hardly be better than chance unless a sensory threshold has been exceeded. However, we must emphasize that our rejection of threshold theory is based on the supposition that were the theory correct, our instructions would have succeeded in getting our Os to use "no's" in Experiment III and "1s" and "2s" in Experiment II only in the nondetect state. About 15% or so of these responses being actually given in the detect state could account for our troublesome findings. Similar reasoning has been used before by proponents of threshold theory to explain away other awkward data (see

Krantz, 1969). The utility of a theory that needs to be rescued in this fashion with such regularity seems debatable.

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(Accepted for publication April 19, 1972.)